 **Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Maths Specialist – Investigation 2016**

**Circle Geometry- Part One**

**Extended investigation Part 1:** **Preparation activity**

**TASK 13: FIBONACCI USING MATRICES**

**Extended Investigation**

**Unit 2.2**

**Topic : Matrices**

**Course-related information**

The concepts and skills included in this investigation relate to the following dot points within the WA Mathematics Specialist syllabus:

2.2.2 define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity, and inverse

2.2.3 calculate the determinant and inverse of 2 × 2 matrices and solve matrix equations of the form AX = B, where *A* is a 2 × 2 matrix and *X* and *B* are column vectors

2.2.8 define and use inverses of linear transformations and the relationship with the matrix inverse

**Background information**

Students should be able to do basic calculations with matrices, including working with inverses and determinants and be able to use their calculator to perform matrix operations. Students should also be able to do proofs by mathematical induction and to calculate using surds. Students should be familiar with the Fibonacci sequence.

**Task conditions**

It is recommended that students be allowed several days to complete part 1 and then be allocated 30 minutes for the in-class validation. Students will not be expected to remember the rules used in part 1 but should be able to apply them. It is recommended students are not permitted to take their investigative material into the validation test.

**Fibonacci using matrices**

**Extended investigation Part 1:** **Preparation activity**

The Fibonacci sequence is the set of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

The Fibonacci sequence is defined as *Fn+2 = Fn + Fn+1* with. *F1* = 1 and *F2 =* 1 where *Fn* stands for the *n*th Fibonacci number.

**Question 1**

It has been conjectured that 

Test the rule for *n* = 1, 2 and 3, for example for *n =* 1, 

**Question 2**

It has also been conjectured that 

(a) Test the conjecture for *n* = 2, 3 and 8.

(b) Assume the conjecture is true for *n* = “*n*” and test the validity for *n* = “*n +1*”.

Write down your conclusion.

(c) Given  then it follows that .

Calculate the determinant of each side of the equation and hence prove that .

(d) Test the rule  for *n* = 2, 3 and 4.

(e) (i) Form a sequence of matrices 

in simplified version, giving the first six terms of the sequence.

(ii) Use your sequence to write down a simplified matrix for.

**Question 3**

(a) Use the matrix method determined in Question 2 to identify F30.

Explain your method clearly.

(b) With your matrix method to identify F30 which other two Fibonacci numbers can be determined?

**Question 4**

Let *M* = then *M n* = 

Using a method called diagonalisation of matrices....

(a) Given *S* = . State the inverse of *S* (= *S-1*) in terms of *a, b, c* and *d*.

(b) Let *D* =  and *M = S* × *D* × *S-1*

Then *M 2 = S* × *D* × *S-1* × *S* × *D* × *S-1 = S* × *D2*× *S-1*

*M 3 = S* × *D* × *S-1* × *S* × *D2*× *S-1= S* × *D3*× *S-1*

Provide an expression for *M n* in terms of *S, D* and *S -1*

(c) Given *D* = , identify expressions for *D2, D3* and *Dn*. Explain your method.

(d) It is known that for **this** matrix, *M* =, the diagonal matrix *D*,

*D* =  and *S* = .

Determine the matrix *S* -1, expressing the elements of the matrix in exact values.

(e) Consider *D* = 

then *Dn* = 

so *M n = S* × *Dn* × *S-1* becomes

*M n* = ××

Show that this expression can be simplified to

*M n* = 

(f) Given use the expressions found in (e) to write down an expressions for *Fn* in terms of .

(g) Determine *F1, F2* and *F3* using the expressions found in (f).

**FOOTNOTE**

By solving the equation *x2 – x –* 1 *=* 0, then the solution is  .

is called the golden ratio.

\*\* Extension: Investigate the golden ratio.

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**Fibonacci using matrices**

**Extended investigation Part 2: In-class validation (23 marks)**

The Fibonacci sequence is the set of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …..

The Fibonacci sequence is defined as *Fn+2 = Fn + Fn+1* with the first two terms equal to one, i.e. *F1* = 1 and *F2 =* 1 and where *Fn* stands for the nth Fibonacci number.

**Question 1** **(3 marks)**

Let *M* = .

It has been conjectured that  i.e. 

Test the rule for *n* = 4.

**Question 2** **(6 marks)**

Given , i.e. *M n* 

(a) Show that the rule works for *n = 3.* (3)

(b) Show that the rule works for *n* = 10. (3)

**Question 3** **(10 marks)**

Given  then it follows that .

(a) Show that . (3)

(b) Test the rule  for *n* = 2 and for *n* = 3. (4)

(c) Assume the rule is true for *n* = “*n*” and prove the validity of the rule for *n* = “*n* +1”.

(3)

(4)

**Question 4** **(4 marks)**

Given *M n* = 

and *M n* =

(a) State the formula for *Fn* the *n*th term of the Fibonacci series in terms of . (1)

(b) Show how to evaluate without using a calculator. (3)

**End of questions**

**Fibonacci using matrices**

**Extended investigation Part 1:** **Preparation activity**

**Solutions**

**Question 1**

|  |
| --- |
| Solution |

**Question 2 (a)**

|  |
| --- |
| Solution  (a) |

**Question 2 (b)**

|  |
| --- |
| Solution  Assume  Test for *n* = “*n+*1” |

**Question 2 (c)**

|  |
| --- |
| Solution |

**Question 2 (d)**

|  |
| --- |
| Solution |

**Question 2 (e)**

|  |
| --- |
| Solution  i =    ii |

**Question 3 (a)**

|  |
| --- |
| Solution  Since |

**Question 3 (b)**

|  |
| --- |
| Solution |

**Question 4 (a)**

|  |
| --- |
| Solution  *S-1* = |

**Question 4 (b)**

|  |
| --- |
| Solution  *M n = S× D n × S -1* |

**Question 4 (c)**

|  |
| --- |
| Solution    *…* |

**Question 4 (d)**

|  |
| --- |
| Solution  *S-1=* |

**Question 4 (e)**

|  |
| --- |
| Solution  Prove ××    ×× |

**Question 4 (f)**

|  |
| --- |
| Solution  *Fn* = |

**Question 4 (g)**

|  |
| --- |
| Solution |

**Fibonacci using matrices**

**Extended investigation Part 2: In-class validation**

**Solutions and marking key**

**Question 1**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| *n* = 4    *∴* the rule works for *n* = 4 | * Substitutes into formula correctly * Multiplies the matrices correctly * Shows the result gives the appropriate Fibonacci numbers | 1  1  1 |

**Question 2 (a)**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| The rule works for *n* = 3 | * Calculates the matrix correctly * Identifies the correct Fibonacci numbers * Makes an appropriate conclusion | 1  1  1 |

**Question 2 (b)**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| But  ∴  Therefore the rule works for *n* = 10 | * Calculates correct matrix * Identifies the Fibonacci terms * Makes an appropriate conclusion | 1  1  1 |

**Question 3 (a)**

|  |  |
| --- | --- |
| Solution  (a)    But  ∴ | |
| Marking key/mathematical behaviours | Marks |
| * Calculates the value of each determinant * States equality | 2  1 |

**Question 3 (b)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * States the realationship for *n = 2* * Checks LHS = RHS * States the realationship for *n = 3* * Checks LHS = RHS | 1  1  1  1 |

**Question 3 (c)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * Correctly defines the equation for *m* * Substitutes for  *m=n + 1* * Provides relevant conclusion | 1  1  1 |

**Question 4 (a)**

|  |  |
| --- | --- |
| Solution  *Fn =* | |
| Marking key/mathematical behaviours | Marks |
| * Correctly identifies formula for *Fn* | 1 |

**Question 4 (b)**

|  |  |
| --- | --- |
| Solution  (b) | |
| Marking key/mathematical behaviours | Marks |
| * Equates the given expression to * Correctly evaluates using the Fibbonacci sequence * Gives the exact value of the expression | 1  1  1 |